# Solution to Assignment 3

**Supplementary Problems**

1. Express the straight line $ax+by = 1$, $a, b > 0$, in polar coordinates. How about $ax+by = 0$ ?

   **Solution.** Let $c = \sqrt{a^2 + b^2}$. Equation is

   $$1 = r(a\cos\theta + b\sin\theta) = rc\left(\frac{a}{c}\cos\theta + \frac{b}{c}\sin\theta\right) = rc\sin(\theta + \alpha),$$

   where $\alpha \in (0, \pi/2)$ satisfies $\sin\alpha = a/c$. In polar coordinates, the straight line is given by

   $$r = \frac{1}{c\sin(\theta + \alpha)}, \quad \theta \in (-\alpha, -\alpha + \pi).$$

2. Express the hyperbola $x^2 - y^2 = 1$ $(y \geq 0)$ in polar coordinates.

   **Solution.** From $1 = r^2(\cos^2\theta - \sin^2\theta) = r^2\cos 2\theta$ we get

   $$r = \frac{1}{\sqrt{\cos 2\theta}},$$

   where $\theta \in (-\pi/4, \pi/4)$.